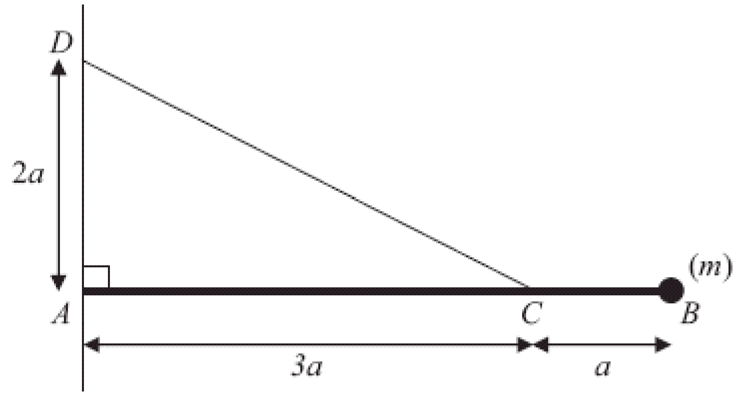
1.



The diagram above shows a uniform rod AB of mass m and length $4a$. The end A of the rod is freely hinged to a point on a vertical wall. A particle of mass m is attached to the rod at B. One end of a light inextensible string is attached to the rod at C, where $AC = 3a$. The other end of the string is attached to the wall at D, where $AD = 2a$ and D is vertically above A. The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is T.

(a) Show that $T = mg\sqrt{13}$.

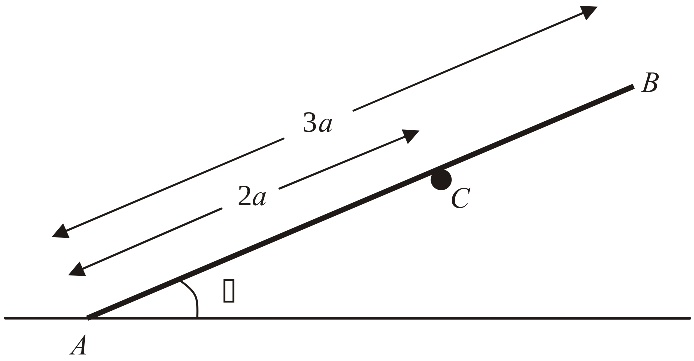
(5)

The particle of mass m at B is removed from the rod and replaced by a particle of mass M which is attached to the rod at B. The string breaks if the tension exceeds $2mg\sqrt{13}$. Given that the string does not break,

(b) show that $M \leq \frac{5}{2}m$.

(3)
(Total 8 marks)

2.



A plank rests in equilibrium against a fixed horizontal pole. The plank is modelled as a uniform rod AB and the pole as a smooth horizontal peg perpendicular to the vertical plane containing AB. The rod has length $3a$ and weight W and rests on the peg at C, where $AC = 2a$. The end A of the rod rests on rough horizontal ground and AB makes an angle α with the ground, as shown in the diagram above.

(a) Show that the normal reaction on the rod at A is $\frac{1}{4}(4 - 3\cos^2\alpha)W$.

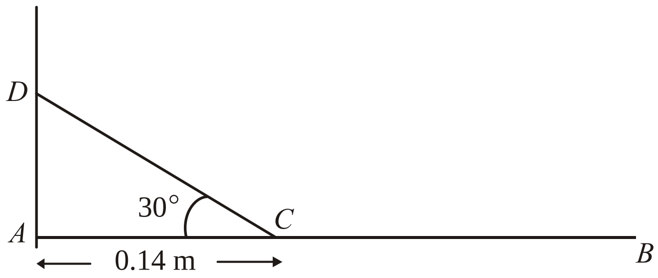
(6)

Given that the rod is in limiting equilibrium and that $\cos\alpha = \frac{2}{3}$,

(b) find the coefficient of friction between the rod and the ground.

(5)
(Total 11 marks)

3.



A uniform beam AB of mass 2 kg is freely hinged at one end A to a vertical wall. The beam is held in equilibrium in a horizontal position by a rope which is attached to a point C on the beam, where AC = 0.14 m. The rope is attached to the point D on the wall vertically above A, where $\angle ACD$ = 30°, as shown in Figure 3. The beam is modelled as a uniform rod and the rope as a light inextensible string. The tension in the rope is 63 N.

Find

(a) the length of AB,

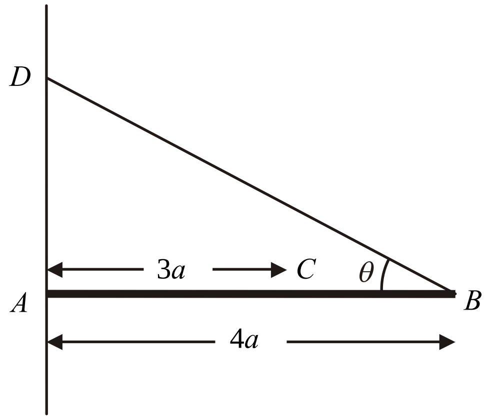
(4)

(b) the magnitude of the resultant reaction of the hinge on the beam at A.

(5)

(Total 9 marks)

4.



A horizontal uniform rod AB has mass m and length $4a$. The end A rests against a rough vertical wall. A particle of mass $2m$ is attached to the rod at the point C, where $AC = 3a$.
One end of a light inextensible string BD is attached to the rod at B and the other end is attached to the wall at a point D, where D is vertically above A. The rod is in equilibrium in a vertical plane perpendicular to the wall. The string is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{3}{4}$, as shown in the diagram above.

(a) Find the tension in the string.

(5)

(b) Show that the horizontal component of the force exerted by the wall on the rod has magnitude $\frac{8}{3}mg$.

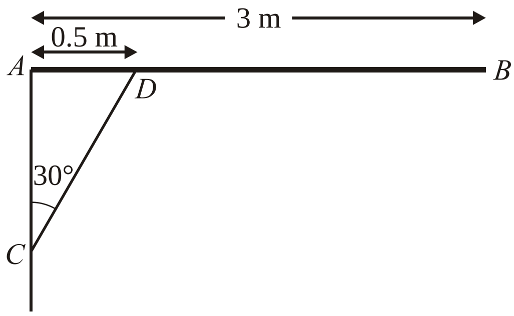
(3)

The coefficient of friction between the wall and the rod is μ. Given that the rod is in limiting equilibrium,

(c) find the value of μ.

(4)
(Total 12 marks)

5.



A uniform pole AB, of mass 30 kg and length 3 m, is smoothly hinged to a vertical wall at one end A. The pole is held in equilibrium in a horizontal position by a light rod CD. One end C of the rod is fixed to the wall vertically below A. The other end D is freely jointed to the pole so that $\angle ACD = 30°$ and $AD = 0.5$ m, as shown in the diagram. Find

(a) the thrust in the rod CD,

(4)

(b) the magnitude of the force exerted by the wall on the pole at A.

(6)

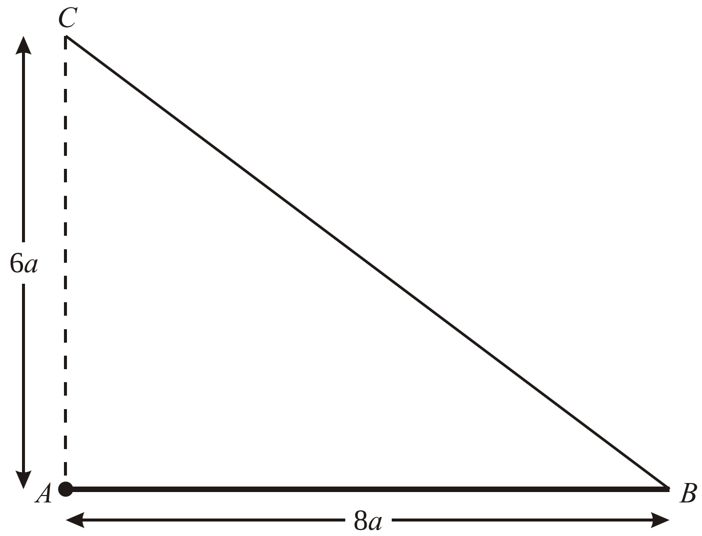
The rod CD is removed and replaced by a longer light rod CM, where M is the mid-point of AB. The rod is freely jointed to the pole at M. The pole AB remains in equilibrium in a horizontal position.

(c) Show that the force exerted by the wall on the pole at A now acts horizontally.

(2)

(Total 12 marks)

6.



A uniform rod AB, of length $8a$ and weight W, is free to rotate in a vertical plane about a smooth pivot at A. One end of a light inextensible string is attached to B. The other end is attached to point C which is vertically above A, with $AC = 6a$. The rod is in equilibrium with AB horizontal, as shown in the diagram.

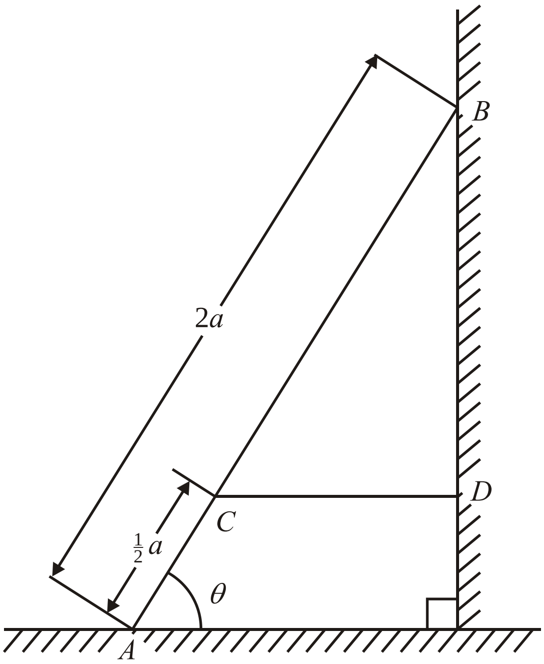
(a) By taking moments about A, or otherwise, show that the tension in the string is $\frac{5}{6}W$.

(4)

(b) Calculate the magnitude of the horizontal component of the force exerted by the pivot on the rod.

(3)
(Total 7 marks)

7.



A uniform ladder, of weight W and length $2a$, rests in equilibrium with one end A on a smooth horizontal floor and the other end B on a rough vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the ladder is μ. The ladder makes an angle θ with the floor, where $\tan\theta = 2$. A horizontal light inextensible string CD is attached to the ladder at the point C, where $AC = \frac{1}{2}a$. The string is attached to the wall at the point D, with BD vertical, as shown in the diagram above. The tension in the string is $\frac{1}{4}W$. By modelling the ladder as a rod,

(a) find the magnitude of the force of the floor on the ladder,

(5)

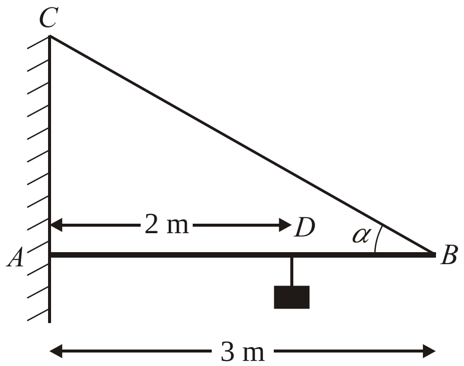
(b) show that $\mu \geq \frac{1}{2}$.

(4)

(c) State how you have used the modelling assumption that the ladder is a rod.

(1)
(Total 10 marks)

8.



A uniform steel girder AB, of mass 40 kg and length 3 m, is freely hinged at A to a vertical wall. The girder is supported in a horizontal position by a steel cable attached to the girder at B. The other end of the cable is attached to the point C vertically above A on the wall, with $\angle ABC = \alpha$, where $\tan\alpha = \frac{3}{4}$. A load of mass 60 kg is suspended by another cable from the girder at the point D, where $AD = 2$ m, as shown in the diagram above. The girder remains horizontal and in equilibrium. The girder is modelled as a rod, and the cables as light inextensible strings.

(a) Show that the tension in the cable BC is 980 N.

(5)

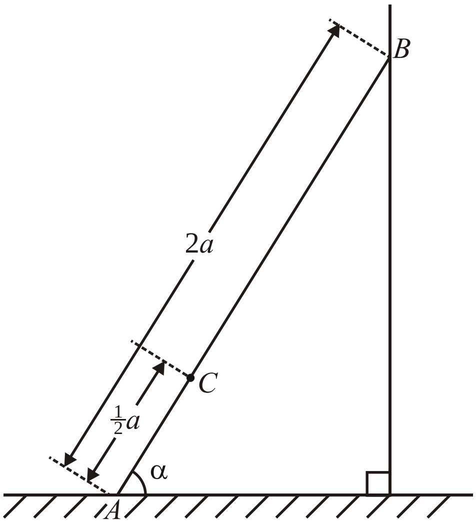
(b) Find the magnitude of the reaction on the girder at A.

(6)

(c) Explain how you have used the modelling assumption that the cable at D is light.

(1)
(Total 12 marks)

9.

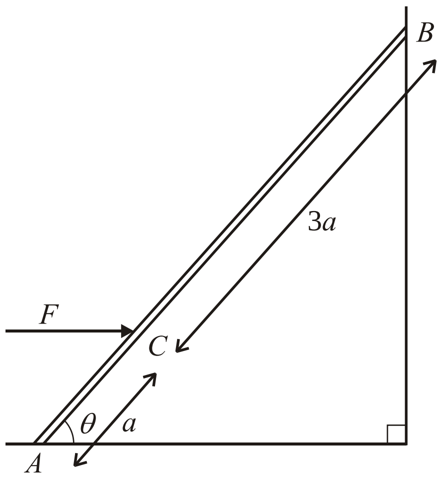


A uniform ladder AB, of mass m and length $2a$, has one end A on rough horizontal ground. The other end B rests against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The ladder makes an angle α with the horizontal, where $\tan \alpha = \frac{4}{3}$. A child of mass $2m$ stands on the ladder at C where $AC = \frac{1}{2}a$, as shown in the diagram above. The ladder and the child are in equilibrium.

By modelling the ladder as a rod and the child as a particle, calculate the least possible value of the coefficient of friction between the ladder and the ground.

(Total 9 marks)

10.

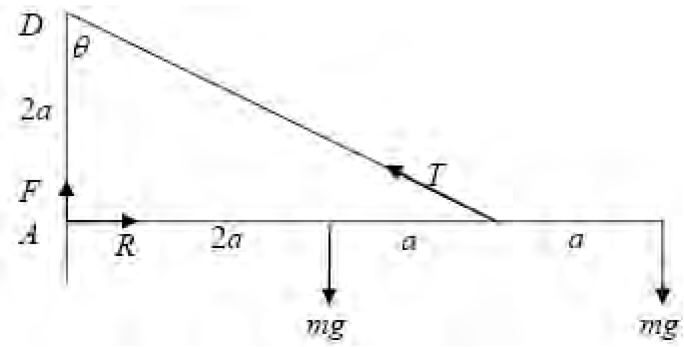


A uniform ladder AB has one end A on smooth horizontal ground. The other end B rests against a smooth vertical wall. The ladder is modelled as a uniform rod of mass m and length $4a$. The ladder is kept in equilibrium by a horizontal force F acting at a point C of the ladder where $AC = a$. The force F and the ladder lie in a vertical plane perpendicular to the wall. The ladder is inclined to the horizontal at an angle θ, where $\tan \theta = 2$, as shown in the diagram above.

Find F in terms of m and g.

(Total 6 marks)

1. (a)



M(A) $3a \times T\cos\theta = 2amg + 4amg$ M1 A1 A1

$\cos\theta = \left(\dfrac{2}{\sqrt{9+4}} =\right)\dfrac{2}{\sqrt{13}}$ B1

$\dfrac{6}{\sqrt{13}}T = 6mg$

$T = mg\sqrt{13}$ * A1 5

(b) $3a \times T \times \cos\theta = 2amg + 4aMg$ M1

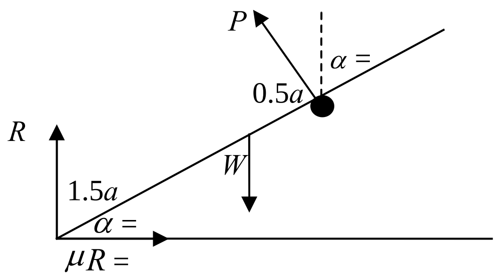
$T = \dfrac{(2mg + 4Mg)}{6}\sqrt{13} \le 2mg13$ A1

$mg + 2MG \le 6mg$

$M \le \dfrac{5}{2}m$ * cso A1 3

[8]

2. (a)



$R(\uparrow)$ $R + P\cos\alpha = W$ M1A1
$M(A)$ $P \times 2a = W \times 1.5a\cos\alpha$ M1A1
$\left(P = \dfrac{3}{4}W\cos\alpha\right)$

$R = W - P\cos\alpha = W - \dfrac{3}{4}W\cos^2\alpha$ DM1

$= \dfrac{1}{4}(4 - 3\cos^2\alpha)W$ * cso A1 6

(b) Using $\cos\alpha = \frac{2}{3}$, $R = \frac{2}{3}W$ B1

$R(\rightarrow)$ $\mu R = P \sin\alpha$ M1A1

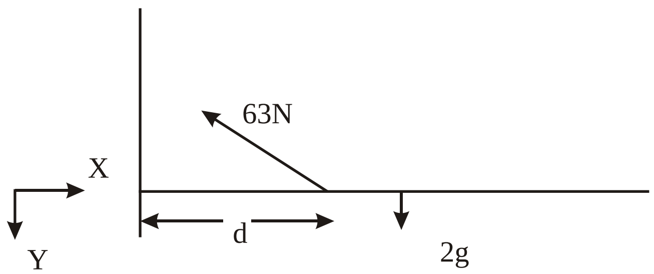
Leading to $\mu = \frac{3}{4}\sin\alpha$

$\left(\sin\alpha = \sqrt{\left(1 - \frac{4}{9}\right)} = \frac{\sqrt{5}}{3}\right)$

$\mu = \frac{\sqrt{5}}{4}$ awrt 0.56 DM1A1 5

[11]

3. (a)



M(A) 63 sin 30 . 14 = 2g.d M1A1A1

Solve: d = 0.225 m

Hence AB = <u>45 cm</u> A1 4

(b) R(→) $X = 63\cos 30$ (≈ 54.56) B1

R(↑) $Y = 63\sin 30 - 2g$ (≈ 11.9) M1A1

$R = \sqrt{(X^2 + Y^2)} \approx$ <u>55.8, 55.9 or 56 N</u> M1A1 5

M1 Take moments about A. 2 recognisable force × distance terms involving 63 and 2(g).

A1 63 N term correct

A1 2g term correct.

A1 AB = 0.45(m) or 45(cm). No more than 2sf due to use of g.

B1 Horizontal component (Correct expression – no need to evaluate)

M1 Resolve vertically – 3 terms needed. Condone sign errors. Could have cos for sin

Alternatively, take moments about B: $0.225 \times 2g = 0.31 \times 63\sin 30 + 0.45Y$
or C: $0.14Y = 0.085 \times 2g$

A1 Correct expression (not necessarily evaluated) – direction of Y does not matter.

M1 Correct use of Pythagoras

A1 55.8(N), 55.9(N) or 56 (N)

OR For X and Y expressed as F cos θ and Fsin θ.

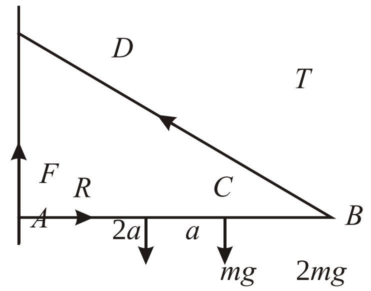
M1 Square and add the two equations, or find a value for tan θ, and substitute for sin θ or cos θ

A1 As above.

N.B. Part (b) can be done before part (a). In this case, with the extra information about the resultant force at A, part (a) can be solved by taking moments about any one of several points. M1 in (a) is for a complete method – they must be able to substitute values for all their forces and distances apart from the value they are trying to find...

[9]

4. (a)



M(A) $T \sin \theta \times 4a = mg \times 2a + 2mg \times 3a$ M1*A1=A1

$T = \frac{8mg}{4} \times \frac{5}{3} = \frac{10}{3} mg$ DM1*A1 5

Accept 32.7m, 33m

Alternative approach:

$\rightarrow R = T \cos \theta$

$\uparrow F + T \sin \theta = 3mg$

M(B) $F \times 4a = mg \times 2a + 2mg \times a (\Rightarrow F = mg)$

$\Rightarrow mg + T \sin \theta = 3mg \Rightarrow T = \frac{2mg}{\sin \theta} = \frac{10mg}{3}$

If they use this method, watch out for F = mg just quoted in (c): M1A1

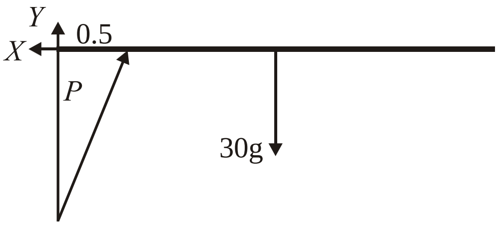
(b) $\rightarrow R + T \sin \theta = 3mg \Rightarrow F = mg$ ft their T M3

Or: M(B) $F \times 4a = mg \times 2a + 2mg \times a \Rightarrow F = mg$ A1ft

(c) $F = \mu R \Rightarrow \mu = \frac{3}{8}$ M1A1 4

[12]

5. (a)



M(A):

$P \times 0.5 \sin 60 = 30g \times 1.5$ M1 A2

$P = 90g.\dfrac{2}{\sqrt{3}} \approx 1020$ N (1000N) A1 4

(b) $\rightarrow X = P \cos 60 = \dfrac{1}{2}P$ N1 A1

($\approx$ 509 N (510N))

$\uparrow$ $Y + P \cos 30 = 30g$ M1 A1

($\Rightarrow Y = -588$ N)

resultant = $\sqrt{(X^2 + Y^2)} = \sqrt{(509^2 + 588^2)} \approx 778$ N or 780 N M1 A1 6

(c) In equilibrium all forces act through a point M1

P and weight meet at mid-point;

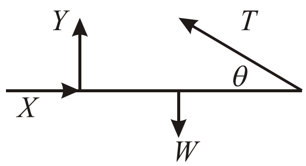
hence reaction also acts through mid-point so reaction horizontal A1 cso 2

OR M(mid-point): $Y \times 1.5 = 0 \Rightarrow Y = 0$ M1

Hence reaction is horizontal A1

[12]

6.



(a) $M(A)$ $W \times 4a = T \times 8a \sin \theta$ M1 A1

Using a value of $\sin \theta$ and solving M1

$T = \dfrac{5}{6}W$ (*) cso A1 4

(b) $\rightarrow$ $X = T \cos \theta$ M1 A1

$= \dfrac{2}{3}W$ A1 3

[7]

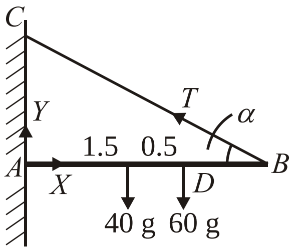
7. (a) $M(B)$, $N\,2a \cos \theta = \mathrm{W}\,a \cos \theta + \dfrac{1}{4}W\,\dfrac{3a}{2}\,\sin \theta$ M1 A2 (–1 e.e.)

$N = \dfrac{7W}{8}$ dep. M1 A1 5

	(b)	$R = \frac{1}{4}W$; $F + N = W$	B1; B1	
		$F \le \mu R$ or $F = \mu R$	M1	
		$\frac{1}{2} \le \mu$ * (exact)	A1 c.s.o.	4
	(c)	It does not bend or has negligible thickness	B1	1

[10]

8. (a)

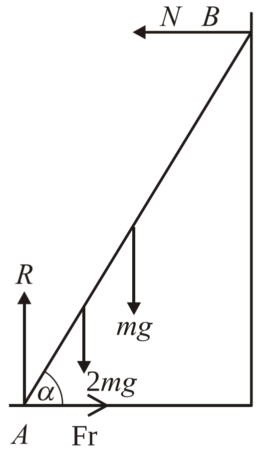


M (A),		
$40g \times \frac{3}{2} + 60g \times 2 = T \sin \alpha \times 3$	M1 A2, 1, 0	
use of $\sin \alpha = \frac{3}{5}$	B1	
$60g + 120g = \frac{9T}{5}$		
$\Rightarrow T = 100g = 980$ N (*)	A1	5

(b)	$(\rightarrow)$: $X = T \cos \alpha$	M1 A1	
	$(\uparrow)$ $Y + T \sin \alpha = 100g$	M1 A1	
	$R = \sqrt{(X^2 + Y^2)} = \sqrt{(784^2 + 392^2)}$	M1	
	= 877 N (3 sf)	A1	6
(c)	Cable light $\Rightarrow$ tension same throughout $\Rightarrow$ force on rod at D is $60g$	B1	1

[12]

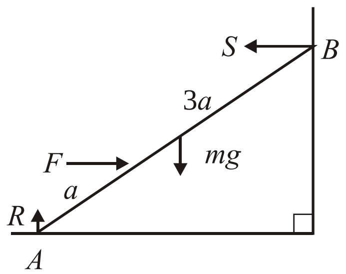
9.



$(\uparrow)$ $R = 3mg$	B1	
M(B)		
$mga\cos\alpha + 2mg \times \frac{3}{2}a\cos\alpha + \text{Fr} \times 2a\sin\alpha = R \times 2a\cos\alpha$	M1 A2 1,0	
Solving to Fr = $\frac{3}{4}mg$	M1 A1	
$\text{Fr} \le \mu R \Rightarrow \frac{3}{4}mg \le \mu 3mg$	M1	
$\mu \ge \frac{1}{4}$ (least value is $\frac{1}{4}$)	M1 A1	9

[9]

10.



R $(\rightarrow)$, $F = S$	B1	
M (A)		
$mg \quad mg\,2a\cos\theta \;+ F\,a\sin\theta \;= S \times 4a\sin\theta$	M1 A2	
i.e. $2mg + 2F = 8S$		
$F = \frac{1}{3}mg$	M1 A1	6

[6]

1. There were many correct responses to this question, some considerably more concise than others.

In part (a) many candidates took the direct route of the mark scheme, and most dealt confidently with the exact trig ratio. There were several who had initially made a false start, resolving vertically and horizontally and ignoring all or part of the reaction at the hinge, but they often went on to score full marks by later taking moments about A correctly. Some did not seem to understand the significance of requiring an exact answer and obtained the given answer in surd form from a decimal value of sin 33.69°.

In part (b) most candidates learned from their experience in (a) and started by taking moments about A. A minority tried several alternative options before deciding to take moments about A. Some candidates did not deal appropriately with the inequality, either by including it when taking moments or by simply inserting it in the final line.

2. This proved to be the most challenging question on the paper. It turned out to be a question which differentiated well between those who had a thorough understanding of the principles and those with a relatively poor grasp. As usual, too many solutions were spoiled by candidates who were not able to find the correct components of their forces, confusing cos α and sin α.

(a) Candidates who took moments about A and resolved vertically were the most successful with many producing compact and complete solutions. Others taking moments about C or B invariably omitted a force or were not able to deal with the extra unknowns.

Common errors were in mistakes with the reactions at A and C, e.g. a vertical reaction at C, the reaction at A perpendicular to the rod, and use of horizontal and vertical components at C with no method for combining them. Several candidates had the weight of the plank acting at C. Also, some introduced a reaction at B, usually horizontal, perhaps through confusion with problems relating to a ladder against a wall.

(b) This was answered a little more successfully with good use of the given answer from part (a). Most candidates seemed confident in their use of $F = \mu R$ although some confused the reaction at A and the reaction at C. This confusion was sometimes the consequence of poorly labelled diagrams. There was some difficulty in finding and using the value for sin α.

3. (a) A few candidates struggled to find their way into this question, sometimes attempting to start by taking moments about B, C or D rather than about A. Weaker candidates would have helped themselves by marking the unknown distance clearly on the diagram. There were however many correct solutions, with most errors due to omitting the distance from a term in the moments equation, or omitting g from the weight.

(b) Many candidates assumed that the resultant reaction on the hinge at A was perpendicular to the wall, or perpendicular to the beam. Candidates who attempted to use moments rather than resolving tended to be more prone to error. Those candidates who did find both components usually went on to combine them correctly to find the reaction. Inappropriate accuracy in the final answer was a common problem – 4s.f. is not appropriate having used an approximation for g.

4. Some candidates were confused about the nature of contact forces and tried to introduce extra forces at either end of the string, but on the whole it was pleasing to see few such errors and many completely correct solutions. Common errors were in incorrect resolution with confusion between sine and cosine components, missing g in the weight terms and occasionally a failure to include distances in moments equations. Some candidates were baffled by the reaction at the wall and its link to friction – their answers suggested a relationship of the form (horizontal force) = μ × (vertical force) rather that an equation linking friction and the normal reaction.

5. This question caused considerable difficulty, even to the better candidates. There was much confusion about what forces were acting where – particularly at the hinge A and at the point D. Some were able to find the thrust in (a) by taking moments about A but often forces were omitted or extra ones included and final answers were rarely given to the appropriate accuracy. The second part was also poorly done with many candidates simply giving either the horizontal or vertical component only. There were very few convincing solutions to part (c), with many simply assuming the answer rather than showing it.

6. This question was very well answered. In part (a) the majority of candidates showed that the principles of moments were well understood and that they knew how to establish the required exact answer. In part (b), a few could not analyse the force at the hinge but the majority were able to find the horizontal component asked for. A few went on to find the vertical component and some then combined the components into a resultant force. The examiners ignore such superfluous work but valuable time had been wasted. Time pressure often results from the use of inefficient methods and from doing unnecessary work rather than from the intrinsic difficulty of the questions.

7. This question was generally well done by the better candidates, with 8 or 9 marks a common score but the weaker ones, many of whom were not clear about the difference between resolving and taking moments, found it very difficult.A substantial number of candidates failed to draw a diagram, leaving examiners to guess the meaning and direction of R, S etc. Few candidates scored full marks. A handful of candidates had friction at the floor and not at the wall.

Even fewer had friction at both the wall and the floor. The two most common errors were to have friction pointing down (not one candidate explained the resulting negative friction) and, most common of all, to use Wg as the weight. Some candidates found angle *è* and so introduced errors into their work. One or two candidates failed to realise that $T = \frac{1}{4}W$.

Generally the moments equation was done well in part (a). Most candidates correctly resolved ? And ? (at the beginning of (a)) and the majority correctly used *f* = *ìr* or *f* ≤*ìr* in part (b). Very few correctly worked through with the inequality. Roughly half the candidates answered part (c) correctly. Weight at the centre was the most common wrong answer.

8. (a) There were quite a few correct solutions with most coming from taking moments about A.

(b) Hinge forces continue to cause problems and a substantial number of candidates don’t seem to appreciate the fundamental point that the direction of a hinge force is not pre-determined – thus many assumed that the force at A was acting either horizontally or vertically which meant that they lost a lot of marks. Many final answers were given to more than 3 significant figure and lost the final mark.

(c) The vast majority scored a mark here.

9. The moments question is often the least successfully tackled on M2 papers but this one was answered substantially correctly by many. It was, however, surprising to see *g* omitted from the weights by so many candidates. Treating mass as a force is a serious mistake and in many questions this would lead to a heavy loss of marks. However in this case the answer is not affected and the general outline of the solution is unchanged. Hence the examiners felt that a loss of two marks was sufficient, effectively treating the error as a misread. The moments equation, whether about *B* or *A*, was generally well attempted. The most common mistake was the omission of one of the forces at the ground in taking moments about *B*. It was disappointing to note that few candidates produced the most elegant solution, which is to find separate expressions for *F* and *R* and then to use $F \leq \mu R$ to obtain $\mu \geq \frac{1}{4}$. The majority of candidates used $F = \mu R$ from the start, apparently not noting that they had not been given that the system was in limiting equilibrium, and only a few attempted to justify why their answer represented the least value of *m*.

10. No Report available for this question.